

# The Causality Principle in the Field Theory of Gravitation

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## Abstract

The causality principle for the Relativistic Theory of Gravitation (RTG) is presented. It is a straightforward consequence of the RTG basic postulates. The necessary conditions for physical solutions of the gravitational field equations to be fulfilled are given.

The Relativistic Theory of Gravitation [1] (RTG) as the field theory of gravitation is based on a hypothesis, that the gravitational field, as well as all other fields, propagates in the Minkowski space, and its source is such a universal conserving quantity as the energy-momentum tensor of all the matter, including the gravitational field itself. This approach allows one to uniquely construct a theory of the gravitational field as a gauge theory (within the framework of second order equations).

A complete set of the RTG gravitational equations in a system of units  $\hbar = c = G = 1$  looks like

$$\gamma^{\alpha\beta} D_\alpha D_\beta \tilde{\Phi}^{\mu\nu} + m^2 \tilde{\Phi}^{\mu\nu} = 16\pi t^{\mu\nu}, \quad (1)$$

$$D_\nu \tilde{\Phi}^{\mu\nu} = 0. \quad (2)$$

Here  $\gamma^{\alpha\beta}$  is a metric tensor of the Minkowski space in arbitrary coordinates;  $\tilde{\Phi}^{\mu\nu} = \sqrt{-\gamma} \Phi^{\mu\nu}$  is a density of the gravitational field tensor;  $D_\mu$  is the covariant derivative of the Minkowski space;  $m$  is a rest mass of the gravitational field;  $t^{\mu\nu}$  is a density of the energy-momentum tensor of all matter.

Density of the matter energy-momentum tensor  $t^{\mu\nu}$  consists of a density of the gravitational field energy-momentum tensor  $t_g^{\mu\nu}$  and a density of the

substance energy-momentum tensor  $t_M^{\mu\nu}$ . We call the substance all the matter fields, except for the gravitational field

$$t^{\mu\nu} = t_g^{\mu\nu} + t_M^{\mu\nu}. \quad (3)$$

The interaction of a gravitational field and a substance is taken into account in the energy-momentum tensor density of substance  $t_M^{\mu\nu}$ . The gravitational field differs from all the known fields in that the gravitational interaction affects the terms with higher (second) derivatives, whereas all other fields do not enter terms with second derivatives. This feature of the gravitational field also gives rise to the effective Riemannian space. Density of the energy-momentum tensor  $t^{\mu\nu}$ , according to Hilbert, can be expressed through the Lagrangian scalar density  $L$  as follows:

$$t^{\mu\nu} = -2 \frac{\delta L}{\delta \gamma_{\mu\nu}}, \quad (4)$$

where

$$\frac{\delta L}{\delta \gamma_{\mu\nu}} = \frac{\partial L}{\partial \gamma_{\mu\nu}} - \partial_\sigma \left( \frac{\partial L}{\partial \gamma_{\mu\nu,\sigma}} \right), \quad \gamma_{\mu\nu,\sigma} = \frac{\partial \gamma_{\mu\nu}}{\partial x^\sigma}. \quad (5)$$

Eqs. (1) and (2) can be derived from the principle of least action only if the Lagrangian density is taken as follows [1]:

$$L = L_g + L_M(\tilde{g}^{\mu\nu}, \Phi_A). \quad (6)$$

Here

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\Phi}^{\mu\nu}, \quad (7)$$

$$\tilde{g}^{\mu\nu} = \sqrt{-g} g^{\mu\nu}, \quad \tilde{\gamma}^{\mu\nu} = \sqrt{-\gamma} \gamma^{\mu\nu}, \quad \Phi_A \text{ — fields of substance.}$$

The Lagrangian density of the gravitational field  $L_g$  in (6) is [1]

$$L_g = \frac{1}{16\pi} \tilde{g}^{\mu\nu} (G_{\mu\nu}^\lambda G_{\lambda\sigma}^\sigma - G_{\mu\sigma}^\lambda G_{\nu\lambda}^\sigma) - \frac{m^2}{2} \left( \frac{1}{2} \gamma_{\mu\nu} \tilde{g}^{\mu\nu} - \sqrt{-g} - \sqrt{-\gamma} \right). \quad (8)$$

Here  $G_{\mu\nu}^\lambda$  is a third rank tensor

$$G_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda - \gamma_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\sigma} (D_\mu g_{\sigma\nu} + D_\nu g_{\sigma\mu} - D_\sigma g_{\mu\nu}). \quad (9)$$

$\Gamma_{\mu\nu}^{\lambda}$  are the Christoffel symbols of the effective Riemannian space,  $\gamma_{\mu\nu}^{\lambda}$  are the Christoffel symbols of the Minkowski space.

The densities of Lagrangians (6) and (8) are the results of initial standings of the theory. On the basis of (6) it is evident, that the motion of substance in the Minkowski space under impact of a gravitational field is reduced to a motion of substance in the effective Riemannian space. Thus, the effective Riemannian space appears as a straightforward consequence of the hypothesis, that a source of a gravitational field is the conserving energy-momentum tensor of matter .

As the gravitational field (for example) in an inertial system is determined in one frame, then according to Eq. (7) the effective Riemannian space is also determined in one frame, but it means, that it has a prime topology. Thus, the effective Riemannian space arising due to the impact of the gravitational field in the Minkowski space always has a prime topology. In the General Theory of Relativity (GRT) the complicated topologies of Riemannian spaces are possible, and therefore atlas of maps is necessary for its exposition. The field notions about gravitation are rather strong, and therefore they do not allow one to derive the GRT equations on their basis.

From Eqs. (6) and (8) and from the principle of least action we see that Eqs. (1) and (2) can be written as follows [1]:

$$R_{\mu\nu} - \frac{m^2}{2}(g_{\mu\nu} - \gamma_{\mu\nu}) = 8\pi \left( T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \right), \quad (10)$$

$$D_{\nu}\tilde{g}^{\mu\nu} = 0. \quad (11)$$

Here

$$T^{\mu\nu} = -2\frac{\delta L_M}{\delta g_{\mu\nu}}, \quad T_{\mu\nu} = g_{\mu\sigma}g_{\nu\lambda}T^{\sigma\lambda}. \quad (12)$$

Eqs. (1) and (2), as well as Eqs. (10) and (11), are covariant under arbitrary coordinate transformations and are form-invariant concerning Lorentz transformations. The last statement means, that they precisely obey the Special Principle of Relativity. According to the RTG, the inertial forces and the forces of gravitation are separated, they are of different nature. If the inertial forces can be removed by a choice of the inertial system of coordinates, the forces of gravitation cannot be eliminated by a choice of a frame even locally. J. Synge wrote on this subject almost half century ago [2]: *“In Einstein’s theory the gravitational field is present or absent depending on the*

*Riemannian tensor being nonzero or equal to zero. This property is absolute; it is in no way related to a world line of an observer.”*

Considering the gravitational field as a physical field in the Minkowski space, we should with necessity observe the Causality Principle in the Minkowski space. Its essence is in the following: For a moving test body in the Minkowski space it is always possible to pick such a frame, in which this body will be at rest, whatever is the nature of forces this movement would be called by, and consequently, the requirement

$$d\sigma^2 = \gamma_{00}(x)(dx^0)^2 > 0, \quad \gamma_{00}(x) > 0. \quad (13)$$

should take place. In Minkowski’s paper “Space and time” [3], published in 1909, this standing was formulated as follows: *“We shall introduce now the following postulate. A substance present at any world point can always be viewed as being at rest with respect to a suitable definition of space and time”*.

But taking into account that in the Minkowski space the test body goes along a geodesic line of the effective Riemannian space by impact of a gravitational field, the following requirement should also be fulfilled

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu > 0, \quad (14)$$

that in a frame where the body is at rest becomes:

$$ds^2 = g_{00}(x)(dx^0)^2 > 0, \quad g_{00}(x) > 0. \quad (15)$$

Thus, for a motion of a test body in the Minkowski space it is always possible to pick such a frame, in which this body is at rest, but simultaneously the requirements of causality (13) and (15) should be fulfilled.

$$\begin{aligned} ds^2 &= g_{00}(x)(dx^0)^2 > 0, \\ d\sigma^2 &= \gamma_{00}(x)(dx^0)^2 > 0. \end{aligned} \quad (16)$$

These causality requirements can be written in the following form [1]:

$$\gamma_{\mu\nu}(x)U^\mu U^\nu = 0, \quad (17)$$

$$g_{\mu\nu}(x)U^\mu U^\nu \leq 0. \quad (18)$$

In writing these requirements we also take into account the opportunity of presence in nature of particles with a zero rest mass, which motion happens

along isotropic geodesic lines. Requirements (16), as well as Eqs. (17) and (18) mean that the causality cone of the effective Riemannian space should be positioned inside the causality cone of a Minkowski space. If the causality cone of the Riemannian space was beyond the causality cone of the Minkowski space, that resulted in the impossibility of picking in the Minkowski space a frame, in which the test body be able to stay at rest. It would mean that a three-dimensional force of gravitation of such a “gravitational field” is impossible to balance by any inertial force, as in this case the following inequality would take place:

$$d\sigma^2 = \gamma_{\mu\nu}(x)dx^\mu dx^\nu < 0. \quad (19)$$

From here it follows, that such a “gravitational field” cannot be presented as a physical field propagating in the Minkowski space. By virtue of the causality requirements Eqs. (17) and (18) the effective Riemannian space will possess an isotropic and timelike geodesic completeness.

The Causality Principle also ensures the existence of a spacelike surface in the Riemannian space, which is only once intersected by any non-spacelike curve, i.e. there is a global Cauchy surface, to set initial physical conditions for this or that problem. In solving the Hilbert-Einstein equations one selects only such solutions, for which the following requirement takes place at any point of space:

$$g < 0, \quad (20)$$

and also for any timelike vector  $K_\nu$  the following inequality is fulfilled:

$$T^{\mu\nu}K_\mu K_\nu \geq 0, \quad (21)$$

and the quantity  $T^{\mu\nu}K_\nu$  for a given vector  $K_\nu$  should form a non-spacelike vector.

In the RTG in solving Eqs. (10) and (11) it is necessary to select only such solutions, which alongside with requirements (20) and (21) also obey the causality requirements (17) and (18). The causality requirements do not follow from the equations, but this is not unusual, as also in electrodynamics the Causality Principle is not a corollary of Maxwell-Lorentz equations, it is introduced in the form of Eq. (13) as a supplement. From Eqs. (10) and (11) it follows, that the test body moves along a geodesic line of the effective Riemannian space. This line is defined by

$$\frac{dp^\nu}{ds} + \Gamma_{\alpha\beta}^\nu p^\alpha p^\beta = 0, \quad ; p^\nu = \frac{dx^\nu}{ds}, \quad ds^2 = g_{\mu\nu}dx^\mu dx^\nu > 0. \quad (22)$$

Here Christoffel symbol  $\Gamma_{\alpha\beta}^{\nu}$  is

$$\Gamma_{\alpha\beta}^{\nu} = \frac{1}{2}g^{\nu\sigma}(\partial_{\alpha}g_{\sigma\beta} + \partial_{\beta}g_{\sigma\alpha} - \partial_{\sigma}g_{\alpha\beta}).$$

According to the RTG, such motion is not free, as it occurs in the Minkowski space under impact of a gravitational field force. A concept of gravitational force is absent in the GRT. J. Synge wrote so related to this subject [2]: “*In the Theory of Relativity the concept of force of gravitation is absent, as the gravitational properties are naturally build in the structure of space-time and are exhibited in a space-time curvature, i.e. in that the Riemannian tensor  $R_{\mu\nu\lambda\sigma}$  is different from zero*”.

In the RTG the concept of force of gravitation is preserved, as the gravitation is obliged to the existence of a gravitational field in the Minkowski space. Below we shall determine this force, basing on Causality Principle (17) and (18) and following Ref. [4]. According to the definition of the covariant derivative in the Minkowski space, we have

$$\frac{Dp^{\nu}}{ds} = \frac{dp^{\nu}}{ds} + \gamma_{\alpha\beta}^{\nu}p^{\alpha}p^{\beta}. \quad (23)$$

By using (22) in (23), we discover

$$\frac{Dp^{\nu}}{ds} = -G_{\alpha\beta}^{\nu}p^{\alpha}p^{\beta}. \quad (24)$$

Let us present the l.h.s. of Eq. (24) in the following form:

$$\frac{Dp^{\nu}}{ds} = \left(\frac{d\sigma}{ds}\right)^2 \left[ \frac{DV^{\nu}}{d\sigma} + V^{\nu} \frac{\frac{d^2\sigma}{ds^2}}{\left(\frac{d\sigma}{ds}\right)^2} \right], \quad V^{\nu} = \frac{dx^{\nu}}{d\sigma}. \quad (25)$$

Here  $V^{\nu}$  is a timelike four-vector of velocity in the Minkowski space. It obeys the following condition:

$$\gamma_{\mu\nu}V^{\mu}V^{\nu} = 1, \quad d\sigma^2 > 0. \quad (26)$$

Substituting (25) in (24), we shall receive

$$\frac{DV^{\nu}}{d\sigma} = -G_{\alpha\beta}^{\nu}V^{\alpha}V^{\beta} - V^{\nu} \frac{\frac{d^2\sigma}{ds^2}}{\left(\frac{d\sigma}{ds}\right)^2}. \quad (27)$$

On the basis of Eq. (26) we have

$$\left(\frac{d\sigma}{ds}\right)^2 = \gamma_{\alpha\beta} p^\alpha p^\beta. \quad (28)$$

By differentiating this expression over  $ds$ , we obtain

$$\frac{\frac{d^2\sigma}{ds^2}}{\left(\frac{d\sigma}{ds}\right)^2} = -\gamma_{\lambda\mu} G_{\alpha\beta}^\mu V^\lambda V^\alpha V^\beta. \quad (29)$$

By substituting this expression in Eq. (27), we shall discover [4]

$$\frac{DV^\nu}{d\sigma} = -G_{\alpha\beta}^\mu V^\alpha V^\beta (\delta_\mu^\nu - V^\nu V_\mu). \quad (30)$$

From here it is obvious, that the motion of a test body in the Minkowski space happens under impact of a four-vector of force  $F^\nu$

$$F^\nu = -G_{\alpha\beta}^\mu V^\alpha V^\beta (\delta_\mu^\nu - V^\nu V_\mu), \quad V_\mu = \gamma_{\mu\sigma} V^\sigma. \quad (31)$$

It is easy to get convinced that

$$F^\nu V_\nu = 0. \quad (32)$$

The l.h.s. of Eq. (30) by definition is equal to

$$\frac{DV^\nu}{d\sigma} = \frac{dV^\nu}{d\sigma} + \gamma_{\alpha\beta}^\nu V^\alpha V^\beta. \quad (33)$$

It should be noted especially that the motion of a test body along a geodesic line of the effective Riemannian space can be understood as a motion in the Minkowski space under impact of force  $F^\nu$ , only if simultaneously the following requirements take place

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu > 0, \\ d\sigma^2 &= \gamma_{\mu\nu} dx^\mu dx^\nu > 0, \end{aligned} \quad (34)$$

i.e. if the Causality Principle is fulfilled.

The force of gravitation and the Riemannian curvature tensor are mutually related. So, if the Riemannian curvature tensor is equal to zero, then

by virtue of Eqs. (10) and (11) the force of gravitation  $F^\nu$  will also be equal to zero. In case the curvature tensor is different from zero, then the force of gravitation is not equal to zero as well. And on the contrary, if the force of gravitation  $F^\nu$  is different from zero, then the Riemannian curvature is not equal to zero either. The vanishing of force of gravitation  $F^\nu$  results in equality to zero of the Riemannian curvature tensor. Due to the impact of a gravitational force  $F^\nu$  there occurs also a fall of a body in a gravitational field, i.e. everything is the same as in Newtonian physics.

Moreover, all gravitational effects in Solar system (deflection of a light beam by the Sun, time retardation of a radiosignal, precession of a gyroscope, Mercury perihelion shift) are caused by impact of the force of gravitation  $F^\nu$ , instead of the Riemannian curvature tensor, which in Solar system is small enough. It is explained by the fact that the force of gravitation is determined by the first derivatives of the metric, whereas the Riemannian curvature tensor — by the second derivatives. As the force of gravitation  $F^\nu$  is a four-vector, it can never be converted to zero by a choice of the frame. Only the three-dimensional part of the force of gravitation  $\vec{F}$  can be converted to zero by a choice of the frame. It also means that the gravitational field as a physical reality cannot be removed even locally.

The similar assertion takes place also for any physical field. A system falling in a gravitational field is not even locally inertial because the motion of a test body along a geodesic line of the effective Riemannian space is not free. If the test body was charged, it would radiate electromagnetic waves, as it goes with acceleration. For this reason the acceleration has an absolute meaning in the RTG, as the concept of inertial systems of coordinates is maintained there.

Authors of article [5] argue that the Causality Principle (17) and (18) is broken for a weak monochromatic wave in the linear approximation. However, this conclusion is incorrect. Below we shall show, that everything is all right with realization of requirements of causality. Eqs. (10) and (11) in the linear approximation of a perturbation theory take the following form:

$$\gamma^{\alpha\beta}\partial_\alpha\partial_\beta\Phi^{\mu\nu} + m^2\Phi^{\mu\nu} = 8\pi T^{\mu\nu}, \quad (35)$$

$$\partial_\nu\Phi^{\mu\nu} = 0. \quad (36)$$

Here  $\gamma^{\alpha\beta} = (1, -1, -1, -1)$ . In the linear approximation neither the interaction of a gravitational field with substance, nor the self-interaction of a

gravitational field are taken into account. The metric tensor of Minkowski space is at higher derivatives in Eqs. (35), and consequently, the requirement of causality for a gravitational field has a standard form

$$d\sigma^2 = \gamma_{\mu\nu} dx^\mu dx^\nu > 0. \quad (37)$$

It is especially noteworthy to mention that the system of equations (35), (36) is physically inconsistent, as according to Eqs. (35) and (36) the energy-momentum conservation law for substance takes place, on the one hand,

$$\partial_\nu T^{\mu\nu} = 0, \quad (38)$$

and, on the other hand, there is a radiation of a gravitational field  $\Phi^{\mu\nu}$ , which with necessity causes losses of substance energy and that contradicts the energy-momentum tensor conservation law for the substance Eq. (38). The effective Riemannian space, which follows from Eqs. (35) and (36), has the following metric

$$g^{\mu\nu} = \gamma^{\mu\nu} + \Phi^{\mu\nu} - \frac{1}{2}\gamma^{\mu\nu}\Phi, \quad \Phi = \Phi^{\mu\nu}\gamma_{\mu\nu}, \quad (39)$$

$$g_{\mu\nu} = \gamma_{\mu\nu} - \Phi_{\mu\nu} + \frac{1}{2}\gamma_{\mu\nu}\Phi, \quad -g = 1 + \Phi. \quad (40)$$

Here  $|\Phi_{\mu\nu}|, |\Phi|$  are small values in comparison with unity. The occurrence of the effective metric of Riemannian space (40) results in the test body (or graviton) moving along a geodesic line of the effective Riemannian space. It follows from the equation of this geodesic line that an integral of motion arises

$$g_{\mu\nu} p^\mu p^\nu = 1, \quad p^\nu = \frac{dx^\nu}{ds}, \quad ds^2 > 0, \quad (41)$$

which takes into account an impact of the gravitational field on a test body (or graviton). In Eqs. (35) and (36) these interactions are not taken into account, and, therefore, there is a mutual coherence between the motion of a test body (or graviton) and the definition of the effective Riemannian metric with Eqs. (35) and (36). On the basis of Eq. (36) we get the following relation for a monochromatic wave

$$p^\mu \Phi_{\mu\nu} = 0. \quad (42)$$

Substituting (40) in (41) and taking into account (42), we get

$$\gamma_{\mu\nu} \frac{dx^\mu}{ds} \cdot \frac{dx^\nu}{ds} = \frac{1}{1 + \frac{1}{2}\Phi} > 0. \quad (43)$$

As for any weak field the value of  $|\Phi|$  is small in comparison with unity by virtue of perturbation theory, it follows from here, that the timelike vector  $p^\nu$  in the effective Riemannian space remains timelike also in the Minkowski space, and consequently, the causality cone of the effective Riemannian space is embedded inside the causality cone of the Minkowski space. Thus, the Causality Principle (17) and (18) is fulfilled also for a weak monochromatic wave.

The introduction of Causality Principle in the form of Eqs. (16) or (17), (18) is not an arbitrary requirement, but with necessity follows as a direct corollary of a hypothesis, that the gravitational field, as well as all other physical fields, propagates in the Minkowski space. Such an idea is implemented in the RTG, and the metric tensor of the Minkowski space is contained in the initial system of equations (10) and (11).

There is an assertion in the literature, that it is possible to present the GRT in the form of a field theory by using the Minkowski space. This is not true. Field notions, as we have already mentioned, with necessity lead to the simple topology of the effective Riemannian space, and also require the Causality Principle in the Minkowski space to be fulfilled. But all these things are not found in the GRT and this is its characteristic feature. Usage of the Minkowski space metric in the GRT is deprived of any physical sense and contradicts the logic of this theory.

Field notions on gravitation together with a hypothesis, that the conserving energy-momentum tensor is a source of the gravitational field, with necessity lead us to the physical conclusion of a nonzero rest mass of the gravitational field, which was reflected in the system of RTG equations. Thus, the presence of a nonzero rest mass of a gravitational field follows from the general enough standings of the theory [1]: The gravitational field is a physical field propagating in the Minkowski space similar to other physical fields, and a source of this field is the universal conserving quantity — the energy-momentum tensor of all the matter including the gravitational field itself.

As there is a general similarity in construction between Maxwell–Lorentz electrodynamics and the RTG, it is natural to assume an opportunity of

existence of a nonzero rest mass also for a photon, as it has been mentioned earlier [6].

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